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## Estimating the Equivalent Initial Crack Size in a Particulate Composite Material under a Multi-Axial Loading Condition

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### Summary

In this study, the equivalent initial crack size,  $a^*$ , in a particulate composite material under a multi-axial loading condition was predicted. The predicted  $a^*$  compares well with the experimentally estimated one. And, the statistical distribution function of  $a^*$  follows the Second Asymptotic Distribution of the Maximum Value.

### Introduction

An important engineering problem in structural design is evaluating structural integrity and reliability. It is well known that structural strength may be degraded during its design life due to mechanical or chemical aging, or a combination of these two aging mechanisms. Depending on the structural design, material type, service loading, and environmental condition, the cause and degree of strength degradation due to the different aging mechanisms differs. One of the common causes of strength degradation is the result of crack development in the structure. When cracks occur, the effects of crack sizes and the rate of growth on the fracture resistance of the material need to be investigated.

The fracture behavior of particulate composite materials has been widely investigated experimentally (1-4). Modeling efforts have been mostly related to correlate crack growth rate to applied stress intensity factor based on the concepts developed by Schapery (5) and Knauss (6) for viscoelastic fracture. In these studies, a deterministic approach was used to develop the crack growth models. Since the particulate composites, on the microscopic scale, can be considered a nonhomogeneous material, a highly nonhomogeneous local stress and strength can be developed in the material. Since the crack growth behavior is controlled by the combination of local stress and strength, it is expected that the crack growth data obtained from a number of tests will show a considerably larger scatter even though the testing conditions remained identical. Under this condition, the statistical method must be used to treat the test data so that the statistical distribution function of the material data can be evaluated and the statistically based mean response and the upper-bound limit can be determined. In addition, in order to predict the service life of a structure, based on crack growth versus time data or a damage-tolerance approach, the initial crack length and its statistical distribution function at zero time need to be determined.

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It is well known in the aerospace industry that the initial flaw size in metals and superalloys is too small to be detected by any nondestructive testing techniques. Consequently, the initial flaw size in metals has been determined using experimental results, such as fractographic data or S-N data. From the experimental S-N data, one can determine the terminal crack size (critical crack size) at the time of failure. Then, the initial flaw size is computed from the terminal crack size by conducting the crack growth analysis backwards (7-8). During the past years, the same approach was used to predict the initial crack size and its statistical distribution function under different strain rate conditions (9). It was founded that the predicted initial crack size and the statistical distribution function were independent of strain rate and the specimen thickness. This indicates that, for a given particulate composite material or for a given material's microstructure, the initial crack size is a material property, and it can be used to develop an inspection criterion to increase the reliability of the structure. In order to prove the initial crack size is a material property, the same material used in Ref. 9 was used to predict the initial crack size under a multi-axial loading condition.

In this study, the equivalent initial crack size (EICS) in a particulate composite material, containing hard particles embedded in a rubber matrix, was determined using fracture and crack growth data generated under a multi-axial loading condition. Uniaxial tensile specimens with and without pre-crack were tested at a constant strain rate of 18.18 cm/cm/min. under 6.9 MPa confined pressure. The results of the analysis are discussed in the following paragraphs.

#### Analytical Analysis

To determine the EICS, the following information is needed: (1) crack growth rate parameters, (2) critical stress intensity,  $K_{IC}$ , and threshold stress intensity factor,  $K_{th}$ , under which a crack will not grow, and (3) time to failure data under constant strain rate. Crack growth rate parameters as well as  $K_{IC}$  and  $K_{th}$  are determined experimentally using pre-flawed specimens. Times to failure data are also obtained experimentally using specimens without a pre-crack.

For pre-cracked specimens, the stress intensity factor,  $K_I$  is given by

$$K_I = \sigma (\pi a)^{1/2} f(a/w) \quad (1)$$

in which  $\sigma$  is the applied stress,  $f(a/w)$  is the geometric correction factor,  $a$  is the crack length, and  $w$  is the width of the specimen. The functional relationship between  $f(a/w)$  and  $a/w$  is shown below.

$$f(a/w) = 0.7722(a/w)^3 + 0.9253(a/w)^2 + 1.0950(a/w) + 1.005 \quad (2)$$

For a specimen subject to a constant strain rate, the stress intensity factor,  $K_I$ , reaches the critical stress intensity factor,  $K_{IC}$ , at the instant of fracture, and the corresponding

flaw size is denoted by  $a_c$ , referred to as the critical crack size or the terminal crack size. It follows from Eq. (1) the

$$K_{IC} = \sigma_c (\pi a_c)^{1/2} f(a_c/w) \quad (3)$$

where  $\sigma_c$  is the critical stress at fracture.

The crack growth rate,  $da/dt$ , has been shown to be a power function of the stress intensity factor,  $K_I$ , i.e.,

$$da/dt = Q K_I^m \quad (4)$$

in which  $m$  and  $Q$  are crack growth rate parameters.

When a specimen without pre-crack is subjected to a constant strain rate loading condition, the entire loading history and hence the stress history,  $\sigma = \sigma(t)$ , can be measured, including the critical stress,  $\sigma_c$ , at the time of fracture,  $t_c$ . For a given critical stress intensity factor,  $K_{IC}$  (material constant), the critical crack size,  $a_c$ , can be computed from Eq. (3). Consequently, the initial flaw size,  $a_0$ , at  $t=0$  can be obtained by integrating Eq. (4), based on the terminal condition ( $a_c$ ,  $t_c$ ) and the stress history,  $\sigma(t)$ .

### Experimental Analysis

Constant strain rate tests were conducted on specimens with and without pre-crack at a strain rate of 18.18 cm/cm/min. The critical stress,  $\sigma_c$ , and the time to failure,  $t_c$ , were determined from the specimen without pre-crack. The crack growth parameters,  $m$  and  $Q$ , were determined from the specimens with pre-crack. The results are:  $m = 2.87$  and  $Q = 0.65 \times 10^{-7}$  in which the units are force in pounds, length in inches, and time in minutes. Further, the critical stress intensity factor and the threshold stress intensity factor are 388 psi (in)<sup>1/2</sup> and 222 psi (in)<sup>1/2</sup>, respectively. In addition, uniaxial edge-cracked tensile specimens with different initial crack lengths (0 in., 0.1 in., and 0.3 in.) were tested at three different displacement rates: 2 in/min, 50 in/min, and 200 in/min.

### Results and Discussion

In the crack growth analysis, the effect of the threshold stress intensity factor for the onset of crack growth,  $K_{th}$ , was not considered. Hence, the flaw size,  $a_0$ , at  $t = 0$  represents the EICS with  $K_{th} = 0$ . By knowing  $K_{th}$ , the time  $t^*$  corresponding to  $K_{th}$  and the crack size at  $t^*$ , denoted by  $a^*$ , can be obtained from the plots of stress intensity factor versus time and crack length versus time. The results of the analysis are shown in Table I

In this study, the equivalent initial crack is a predicted crack assumed to exist in the material. It characterizes the equivalent effect of an actual initial crack in the material. The equivalent initial crack is not a physically observable initial crack. Therefore, the

predicted equivalent initial crack must be justified using applicable test data. In other words, the predicted EICS needs to be verified experimentally. To achieve this goal, uniaxial edge-cracked tensile specimens with different initial crack lengths (0 in., 0.1 in., 0.3 in.) were tested at three different displacement rates (2 in/min, 50 in/min, and 200 in/min). The test results, plotting the maximum stress,  $\sigma_{max}$ , versus the corresponding time,  $t_{max}$ , are shown in Fig.1. By shifting the un-precracked specimen data vertically downward until they superpose upon those of the pre-cracked specimen, we can obtain an estimate for the initial flaw size in the un-precracked specimen. The dashed lines in Fig.1 represent the vertically shifted curves. According to Fig.1, the initial crack size in the un-precracked specimen is approximately equal to 0.1 in., which compares well with the predicted value of 0.116 in. This indicates that the accuracy of the crack growth model and the developed EICS predictive model are excellent.

In addition to determining the EICS, the statistical distribution function of the EICS is also determined. The distribution of initial crack size provides information for determining the threshold crack size for nondestructive inspection. Also, the determination of the size of the initial crack in the particulate composite material may provide information regarding the applicability of using fracture mechanics to predict the crack growth behavior in the material.

In this study, four statistical distribution functions, (1) normal distribution, (2) two-parameter Lognormal distribution, (3) two-parameter Weibull distribution and (4) second asymptotic distribution of maximum values, were considered. A typical plot of the statistical distribution of the second asymptotic distribution of maximum value for  $a^*$  is shown in Fig.2. For a comparison purpose, experimental data, shown as circles, are also included in these figures. It is seen that the Second Asymptotic Distribution of the Maximum Value fits the experimental data very well. In addition, the goodness of fit for different distributions was conducted

using the Kolomogorov-Smirov test. The results also indicate that the Second Asymptotic Distribution of the Maximum Value has the best fit for the distribution of  $a^*$ .

### Conclusion

In this study, the equivalent initial crack size,  $a^*$ , in a particulate composite material subjected a constant strain rate of 18.18 cm/cm/min under 6.9 MPa confined pressure was determined. The results of analysis reveal that the equivalent initial crack size is 0.116 in., which compared well with the experimentally estimated value of 0.1 in., and the statistical distribution function of  $a^*$  follows the Second Asymptotic Distribution of the Maximum Value.

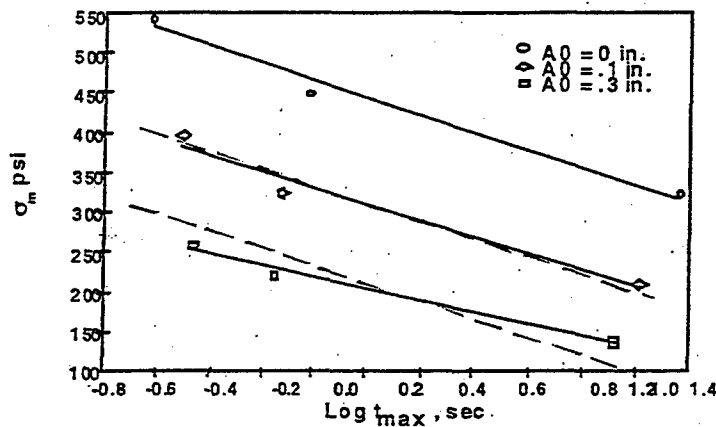


Fig. 1 Maximum stress versus maximum time.

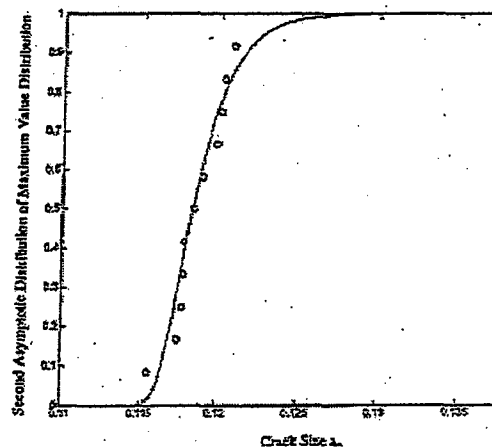


Fig.2 Second asymptotic distribution of the maximum value plot for  $a^*$ .

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Table 1 Summary of Crack Lengths

Specimen	$a_c$	$t_c(\text{sec})$	$a^*$	$t^*(\text{sec})$	$a_0$
Specimen 1	0.12630	1.39630	0.11483	0.80571	0.11373
Specimen 2	0.12717	1.42730	0.11467	0.79640	0.11353
Specimen 3	0.13060	1.52430	0.11909	0.93127	0.11803
Specimen 4	0.13141	1.45830	0.11932	0.84500	0.11820
Specimen 5	0.12571	1.43770	0.11298	0.79527	0.11184
Specimen 6	0.12586	1.40720	0.11406	0.80476	0.11297
Specimen 7	0.12746	1.38830	0.11564	0.78621	0.11452
Specimen 8	0.12921	1.37600	0.11755	0.78270	0.11644
Specimen 9	0.12995	1.43480	0.11771	0.81673	0.11658
Specimen 10	0.12701	1.44890	0.11498	0.83447	0.11387